

**A NEW APPROACH TO ANALYZING UNIVERSITY PRESTIGE AND INTERNAL  
RESOURCE ALLOCATION: *Geometric Interpretations and Implications***

June 2012

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**ABSTRACT**

The content of this paper is based solely on the original study reported in Abe and Watanabe (2012a) in which the authors develop a structured and innovative theory of optimal resource allocation and prestige maximization for an institution of higher education. Although the study provides a sound diagnostic framework and unambiguous results, the paper requires relatively advanced knowledge of mathematics and microeconomic theory, which presents an unattractive feature for students and researchers in academic disciplines that do not necessarily incorporate such analytical methodologies. The primary objective of this paper is placed on presenting some of the theoretical findings demonstrated in Abe and Watanabe (2012a) and the significance of their implications, without sacrificing the original intent of the study, to an audience of non-mathematical users. In doing so, particular emphasis is placed on geometric interpretations and implications rather than reiterating the results with heavy reliance on specialized knowledge of mathematics and economics. Thus, this paper is designed with the aim of conveying informative insights and enhancing solid understanding of the new analytical approach proposed by the authors to a broader audience of students and scholars whose research interests cover the cost effectiveness in financing higher education institutions.

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Growing literature, both academic and nonacademic, on university prestige and rankings exists in sheer reflection of the non-negligible impacts of currently influential worldwide rankings on multiple aspects of today's institutional management and strategy. The scholarly contributions in the literature which has modestly grown over the last few decades, perhaps at a slower rate than the attention paid by the nonacademic and external stakeholders, may generally be categorized into two types of intellectual works. The first group of studies empirically examine the determinants of university rankings or prestige with applications of statistical or econometric methods to existing micro datasets (Brewer *et al.*, 2001; Melguizo and Strober, 2007; Cyrenne and Grant, 2009). The second group typically employs game theoretic approaches and examines the predicted outcomes of strategic actions presumably made by universities and colleges in pursuit of gaining larger market shares in today's multifaceted industry of higher education (Dearden *et al.*, 2008; Del Rey, 2001; Vanhaecht and Pauwels, 2005).

The industry of higher education has been formulated into analysis, particularly by education economists, as a marketplace where individual colleges and universities attempt to maximize institutional prestige or reputation as a result of, or in a more extreme and restrictive sense, in exchange for, provision of multiple products and services such as student instruction, research outputs, as well as community services for their stakeholders which include students, alumni, donors, communities, and governments (Baumol *et al.*, 1982; Breneman, 1976; Brewer *et al.*, 2001; Cohn *et al.*, 1989; Cyrenne and Grant, 2009; Del Rey, 2001; James, 1990; Massy, 1996; Melguizo and Strober, 2007). Despite this vital presumption that every institution of higher education is naturally a prestige

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seeking entity, no scholarly contribution has been made on concretely formulating the process by which these institutions strive to acquire prestige or reputation. Moreover, none has questioned the existence of the "maximized" prestige as well as the required conditions and characteristics if such a state ever exists. Perhaps more importantly, it is unclear whether or not an internal allocation arrangement exists, which "optimally" and "uniquely" maximizes the prestige of an institution. Viewing the same issue from an alternative perspective, what are the potential consequences if such an allocation arrangement does not exist? How could a college or university achieve the "highest" prestige by "optimally" allocating its limited resources? It is on these issues that Abe and Watanabe (2012a) take a theoretical but innovative approach to find solid answers without subjective ambiguity.

This paper provides geometric interpretations of the theoretical findings reported in the original study by the authors, with respect to the optimizing allocation arrangement of financial resources in relation with prestige maximizing behaviors of colleges and universities. Although the authors' findings reveal an interestingly systematic and clear mechanism which helps us understand the relationship between the deliberately allocated resources and the corresponding prestige attained by an institution of higher education, the original paper requires relatively advanced knowledge of mathematics and microeconomics, which is an unappealing feature to involve a wider audience of higher education in research, not to mention those in practice. Furthermore, building on the same analytical foundation and diagnostic tools, the authors also demonstrate that different funding schemes could yield different impacts on the extent to which functional differentiation is achieved by colleges and universities (Abe and Watanabe, 2012b). In addition, the authors further examine the impeding effect of interdisciplinary fields of research and learning on enhancement of functional differentiation (Abe and Watanabe, 2012c). The latter two studies by the authors essentially require firm understanding of the theoretical framework established in the original analysis, and we believe that this heuristic and non-mathematical version could serve as a complementary, but not fully substituting, manual to read along with the original paper. Thus, the content of this paper is designed particularly with the intent of presenting geometric interpretations of the theoretical components as well as significance of their implications without heavily relying on mathematical notations and formulas.

### Colleges and Universities as Prestige Maximizers

A formal and theoretical foundation for studying the optimizing arrangement of internal resource allocation, particularly in relation with existence (or nonexistence) of the highest potential prestige attainable by an institution of higher education was originally formulated by Abe and Watanabe (2012a). The study provides an analytical framework and apparatuses necessary for a fundamental analysis of the optimizing allocation behaviors many colleges and universities are predicted to take in pursuit of institutional prestige under a hypothetically simplified environment. One of the crucial features in the hypothetical setting proposed by the authors is that each institution of higher education under study is assumed to face no severe market rivalry. Certainly, the non-competitive feature is a highly debatable and unattractive presumption to include, particularly when the goal of study is set on examining the strategic behavioral patterns and the state of the reached (or not) equilibrium among the top-tier elite universities that often engage in direct competition with each other to gain larger market shares in terms of acquisition of renowned scholars, high quality students, research grants and funding, *etc.* For the very aim of examining competing strategies and the predicted outcomes for colleges and universities, an application of game theory appropriately serves its purpose, and preceding studies exist which take this analytical approach (Dearden *et al.*, 2008; Del Rey, 2001; Vanhaecht and Pauwels, 2005).

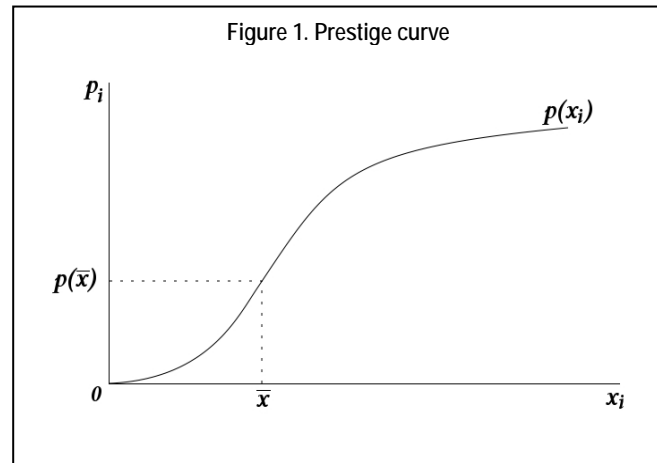
In contrast to the preceding research, the primary focus of Abe and Watanabe (2012a) is placed on examining the predictable outcomes of "ordinary" institutions serving a more general population with limited financial resources, with regard to their efforts in reaching the highest potential prestige which may alternatively be termed "reputation", "social visibility" or "recognition".<sup>1</sup> In other words, the proposed model provides predictable solutions on how an institution of higher education maximizes its prestige by best allocating the available resources in hand, in absence of severe market competition. The hypothetical arrangement may not fully reflect the ruthless reality observed in the high-end market of the industry, in which a few "superstar" universities with enormous endowment expend abundant resources and strive to win a larger piece of a pie. However, such a scenario does not always present a realistic picture of many public institutions, whose mission has been set more or less on serving the immediate needs of surrounding communities. For instance, increasing attention is being paid by state-funded colleges and universities with multiple campuses in the US to accountable and cost-effective service provision of teaching, research, and services to neighboring communities (Nelms *et al.*, 2005) rather than concentrating their efforts and limited resources on acquisition of larger shares in severely competitive outer markets.<sup>2</sup> Nevertheless, such non-competing institutions may also gain or lose their institutional prestige or reputation, depending on the demonstrated outputs that are sternly assessed by external stakeholders. Thus, it is understood that the fundamental motive for individual colleges and universities to seek higher prestige or reputation may well be driven by potential liabilities and pressures imposed by the directly involved stakeholders even without severe market rivalry.<sup>3</sup>

### A Notion of "Prestige Function" and "Prestige Curve"

In order to systematically formulate the analytical procedure, Abe and Watanabe (2012a) introduce a notion of "prestige function" which serves as the basic apparatus to facilitate their theoretical development. Grounded on this simplified and hypothetical tool, but consistently so with our intuition and general observations in reality, the authors build a theoretical model which permits us to

"foresee" the likely consequences of a college or university in its attempt for finding the best allocation arrangement and correspondingly realized institutional prestige.

The authors assume that the prestige of a higher education institution, which is denoted as  $p$ , is an increasing function of financial resources denoted as  $x$ . The assumption simply asserts that the level of institutional prestige, denoted as  $p(x)$  in a combined form, depends on the amount of expendable resources  $x$  or "input". A larger amount of institutional funding certainly enables attainment of higher prestige, whereas a smaller amount of available resources constrains the height of prestige achieved by an institution. It should be clear, at least intuitively, that more resources make it possible for a college or university to reach a higher level of institutional prestige as more money attracts higher quality of scholars, researchers, and students. Therefore, the shape of the prestige curve should be depicted with a positive slope when it is plotted with  $p(x)$  on the vertical axis and  $x$  on the horizontal axis, as shown in Figure 1.



Another notable aspect of the curve in the figure is that the rate of increase in the prestige steadily declines after a certain point so that the level of prestige eventually reaches a plateau even though the amount of financial input  $x$  continues to increase. Intuitively, this portion of the prestige curve indicates that the acquisition of additional units of prestige becomes increasingly harder as the institutional scale expands. A possible rationale behind this phenomenon is explained, for example, by tapering economy of scale due to rising complexities in the internal distribution process of resources, *e.g.*, conflicting interests among different divisions and departments, as an organization grows in its size and scope.<sup>4</sup>

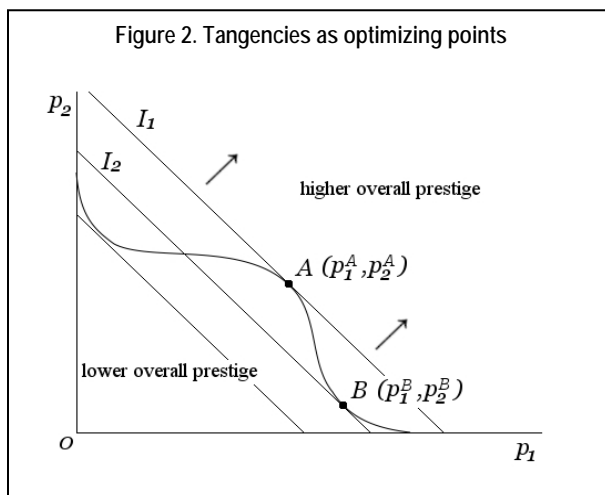
Although the term "prestige" is deemed conceptually similar with the typical "rank" of a college or university, it is clearly not measurable with certainty, or is extremely hard to do so without subjective biases, in a similar sense that different ranking authorities currently use different measuring components, algebraic weights, and formulas with varying subjective reasoning and justifications. The theoretical model proposed by Abe and Watanabe (2012a) does not require accurate measuring of the prestige in a "cardinal" or numerical scale, but it only conceives the prestige as measurable in an "ordinal" manner. Thus, an element critically required for building of the prestige function is the ability to only rank the prestige of an institution, *i.e.*, higher or lower, realized as a result of varying amounts and allocation arrangements of resources. It is highly important to understand that the proposed model and derived results depend only on these fundamental characteristics, which simply regulate the shape of the prestige curve revealing strictly increasing prestige along with the financial resource input, but it does so at a decelerating rate as the institution moves along the prestige curve towards larger scales.<sup>5</sup>

### Solving an Optimization Problem: A Geometric Interpretation

A technical difficulty with the prestige function  $p(x)$  as prescribed thus far is the inability to expand its dimensions; that is, how do we incorporate multiple dimensions of activities and services typically provided by an institution of higher education? For instance, a university offers academic degree programs in multiple disciplines, *i.e.* economics, law, physics, *etc.*, as well as at different levels, *i.e.*, undergraduate, master's and doctoral, for each of which an institution gains separately assessed prestige or reputation. Moreover, a college or university engages not only in student instruction, but it typically engages in research, community services, and collegiate athletic activities. The authors incorporate this multiple dimensionality by (conceptually) summing up separate units of prestige earned in multiple disciplines and institutional activities. That is to say, overall institutional prestige is hypothetically measured in a form of a simple summation  $p_1(x_1) + p_2(x_2) + \dots + p_n(x_n)$ , where  $p_i(x_i)$  is the earned prestige in the  $i$ th academic discipline or functional activity.<sup>6</sup> For a heuristic purpose and presentational convenience which only requires a two-dimensional image rather than larger dimensions, *i.e.*, more than three dimensions which do not allow an easy graphic presentation, we consider in this paper a case where  $n = 2$ ; *e.g.*, a college with two dimensions of institutional functions such as undergraduate and graduate teaching. Alternatively, a hypothetical setting may include two institutional functions such as teaching in general versus research or two specific academic disciplines, *e.g.*, economics versus physics. Then, the situation may graphically be presented as shown in Figure 2 below.

The diagonal lines in the figure are called the “iso-prestige” lines, each of which represents the equal or constant level of overall prestige ( $\bar{P}$ ) earned by an institution with undergraduate ( $p_1$ ) and graduate ( $p_2$ ) education programs.<sup>7</sup> Any combinations of prestige ( $p_1, p_2$ ) in the northeast direction indicate higher levels of combined prestige than the points in the southwest direction. The level of prestige given by the outer iso-prestige line ( $I_1$ ) in Figure 2 certainly requires a larger amount of financial resources than the prestige given by the inner iso-prestige ( $I_2$ ) as the attainment of higher prestige requires more money, as depicted in Figure 1 of the previous section.

An iso-prestige line  $I$  is formulated by a typical linear relation,  $\bar{P} = p_1 + p_2$ , for which a simple algebraic transformation gives  $p_2 = \bar{P} - p_1$ . Since the overall prestige  $\bar{P}$  is treated as constant on each iso-prestige line, the prestige of the graduate program ( $p_2$ ) is solely and inversely related with the level of prestige earned in the undergraduate program ( $p_1$ ). The negative slope (with a coefficient -1) of the iso-prestige lines in the graph clearly represents the inevitable tradeoff between the undergraduate and graduate programs; an increase in the prestige of one program comes at the expense of the other, holding the level of overall prestige  $\bar{P}$  constant. Therefore, the negative slope simply shows the counter-relation that a college or university cannot simultaneously increase the prestige in both undergraduate and graduate programs for a given (equal) level of achievable prestige. It is important to keep in mind that the level of combined prestige,  $\bar{P} = p_1 + p_2$ , is equal along the same iso-prestige line.



The winding curve drawn in Figure 2 is what the authors call the “possible prestige” curve. The curve plots the locus of all the possible combinations of prestige ( $p_1, p_2$ ) separately gained from two different programs, for a given amount of institutional funding  $\bar{X} = x_1 + x_2$ . Fully understanding the specificity of the shape and characteristics of a possible prestige curve, *i.e.*, why the curve has such a shape as shown in the figure, involves mathematics and thus is omitted. However, it should be relatively obvious that an organization with a fixed amount of financial resources can achieve higher joint prestige, *e.g.*, ( $p_1^A, p_2^A$ ) at point A, through careful and efficient allocation of the limited resources than otherwise achieved, *e.g.*, ( $p_1^B, p_2^B$ ) at point B, as a result of wasteful and inefficient internal resource management.

What is important for the purpose of solving an optimization problem here is to understand that the overall institutional prestige, which is formulated as the simple algebraic sum  $p_1 + p_2$ , is maximized where the straight iso-prestige line and the possible prestige curve are attached with each other. Two of such “tangent” points are shown in Figure 2. Again, any points in the northeast direction of an iso-prestige line indicate the points of higher overall prestige. Therefore, point A (tangent with iso-prestige  $I_1$ ) gives a higher level of institutional prestige than point B (tangent with  $I_2$ ). In fact, point B is understood to “locally” (meaning “in the neighborhood of point B”) minimize the overall institutional prestige. The optimizing technique often gives solutions where the objective (*i.e.*, prestige) function is both maximized and minimized. This is true because some of the basic conditions mathematically required to satisfy the optimality are commonly shared by both the maximizing and minimizing points. What is of our interest now is, of course, the maximized prestige given by point A, and a minimizing point B should automatically be disregarded.

Although the described example provides a heuristic and convenient case with only two dimensions of academic teaching, *e.g.*, undergraduate and graduate programs, the situation may well be replaced with two institutional functions, *e.g.*, teaching versus research, or two academic disciplines. Furthermore, the currently restrictive two-dimensional analysis may be extended to examine a more general case with  $n$  fields of institutional functions and academic disciplines, *e.g.*, undergraduate and graduate programs in economics, computer science, and physics, with various faculty research activities, community services and collegiate athletic teams. Unfortunately, demonstrating such a general case cannot be presented with graphical ease due to its multiple and higher dimensionality and so goes beyond the intent of this non-mathematical version. However, a generalization of the two-dimensional analysis presented in this paper is with no doubt feasible and has been conducted by the authors. Interested readers are referred to a more detailed analysis by Abe and Watanabe (2012a). Building on the technical and environmental settings introduced thus far, the authors present two main findings with regard to the optimizing arrangement of internal resource allocation by which an institution maximizes its prestige (or fails to do so otherwise in some extreme conditions).

**Two Main Findings: Geometric Interpretations and Implications**

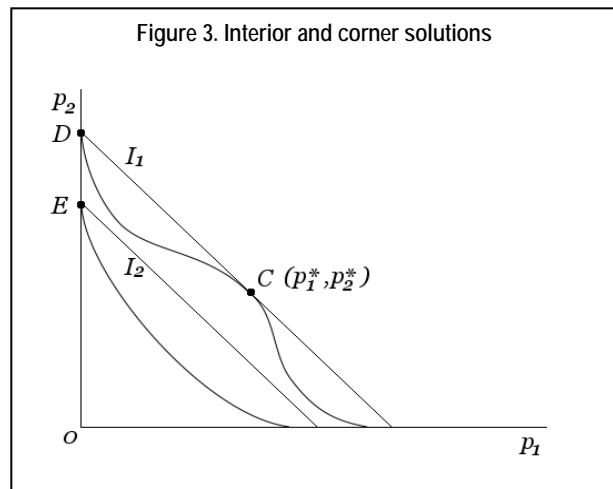
In this section, two theorems proved in the original study by the authors are presented with some intuitive and geometric interpretations. Although the formal proofs of these theorems are omitted, the emphasis is placed on their implicational significance.

**Theorem 1.** *Given a sufficiently small amount of financial resources, there exists no internal allocation arrangement which maximizes the institutional prestige.*

The statement may be surprisingly puzzling at first. It asserts that the allocation arrangement which optimally enables an institution to achieve the maximal prestige does not exist (even though the institution can expend all the available, but limited, resources). In other words, Theorem 1 guarantees that a situation exists, in which a college or university fails to find an optimizing allocation arrangement when it faces a sufficiently severe financial constraint. Abe and Watanabe (2012a) particularly demonstrate two possible scenarios in this regard, one of which leads to a consequence (1) an institution may be able to locate the highest potential prestige by scaling down its size or scope, and (2) an extremely severe financial restraint enables an institution to achieve the highest prestige only by offering a single functional activity, e.g., offer only undergraduate teaching rather than offering both undergraduate and graduate programs. Although the mathematical proof is omitted in this paper, the logic of the proved theorem may be understood with the analytical tools introduced so far without the usage of specialized knowledge of mathematics.

The first scenario describes an organizational environment in which an institution faces a “moderate” financial constraint. The term “moderate” is not meant to give a specific amount, of course, but is meant to give the level of financial resources by which an institution is still able to find the highest prestige (at least locally) by reducing the institutional scale and/or functional scope. The first theorem simply suggests that a concentrated or more focused use of resources for a narrower set of institutional functions enables an acquisition of higher prestige or reputation than maintaining a wider range of meagerly-funded functional activities when an institution faces a sufficiently significant and permanent funding cut. For instance, a tangency at point C in Figure 3 describes such a combination of prestige  $(p_1^*, p_2^*) = (p_1(x_1^*), p_2(x_2^*))$  realized with the total resources  $\bar{X}$  which is allocated between the two activities as  $(x_1^*, x_2^*)$ .<sup>8</sup>

The second scenario proved by the authors provides an extreme case where an institution is forced to retain only a single functional activity, in order to achieve the potentially highest overall prestige. For instance, a college with a severe budgetary constraint, due to a significant funding cut, may be able to attain higher prestige by focusing on a single program than thinly distributing the limited resources between two separate institutional functions, e.g., undergraduate and graduate schools. If this is the case, the best allocation strategy for this institution would be to concentrate its available resources into a single activity rather than holding two institutional functions. Figure 3 depicts such an extreme case where an optimizing allocation has shifted to point E, where the possible prestige curve crosses the iso-prestige line on one side of the axes, a point normally called a “corner solution” in economic theory. The authors mathematically prove that there exists the level of financial resources at which the possible prestige curve becomes a simple convex curve to the origin, as drawn in the graph.



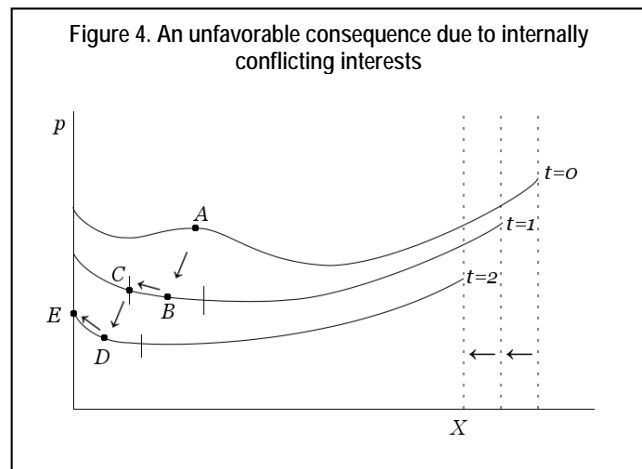
Any combinations of prestige  $(p_1, p_2)$  other than point E in the “interior”, e.g., any points along the convex curve in Figure 3, are located inside (southwest) of the iso-prestige line  $I_2$ , which certainly implies lower levels of overall joint prestige. Therefore, the institution in the second scenario is able to maximize its prestige as a “graduate university” rather than offering both undergraduate and graduate programs. The drastic shift from the interior point C to the corner solution E means a complete closure of an academic program or department. However, the situation reveals that a higher level of prestige is obtained apparently by the concentrated use of resources than any other possible combinations of mixed allocation between two academic programs. Therefore, the first theorem affirms the existence of a situation where no mixed allocation arrangement cost-effectively achieves the maximized prestige or reputation when an institution is constrained by severely limited funding sources.

It is important to notice, however, that under the first scenario an alternative optimizing solution may be sought at point D other than point C, at which a corner solution is also obtained. However, every institution typically has some degree of internal “rigidity” in terms of reallocating financial resources particularly at the time of severe institutional funding cut. Abe and Watanabe (2012a)

systematically integrate this institutional rigidity or “mobility” into the current analysis. The mobility simply incorporates the organizational flexibility with regard to a decision making process in reshuffling resource allocation and is affected by competing divisions or departments within the same institution, due to their internally conflicting interests (Johnson and Turner 2009, Massy 1996). Owing to this confined mobility, the authors discuss that an immediate and drastic jump from the tangency point C ( $p_1^*, p_2^*$ ) to the corner solution D ( $0, p_2^*$ ) is an unlikely, though absolutely possible, movement to make for an institution of higher education.

Finally, a noteworthy scenario predicted as a result of Theorem 1 is an unfavorable consequence of the organizational rigidity on the level of prestige attainable by an institution. As explained above, the internal rigidity limits the ability of an institution to freely reallocate its resources among multiple divisions and/or departments. Figure 4 indicates the process and potential consequence of such an internal conflict. Assume that the institution is originally (at  $t = 0$ ) able to locate the allocation arrangement which locally maximizes the prestige at point A. (Notice that Figure 4 is a “rotated” version of Figure 3 to clearly show the vertical height of the overall prestige and include the amount of available resources in the picture.)

The authors prove that such a maximum point could disappear in the following time period ( $t = 1$ ) if the institutional funding is significantly reduced. However, a drastic reallocation of resources is unlikely as separate divisions and departments may not be able to reach a mutually agreeable allocation arrangement especially if the reallocation is deemed unfairly disproportionate for the involved parties. Thus, the new allocation arrangement may perhaps be sought within the neighborhood of point B (within the permitted range), at a point such as C in Figure 4 where the prestige is the highest locally. If the institutional funding is further reduced in the fiscal year  $t = 2$ , the school eventually moves to point E, via readjustment from point D, at which the potential prestige is heightened as a corner solution. It is important to notice here that the institution could have reached the right-most corner solution which yields a higher level of institutional prestige than point E. As outlined in the progressive manner in Figure 4, however, an institution may end up with the unfavorable consequence with potentially lower prestige, depending on (1) the location of the original allocation arrangement, (2) the extent to which the institution is able to reallocate its resources, that is, the institutional mobility, and (3) the shape of the prestige curve. In fact, this readjustment process by an institution is also drawn as part of Theorem 2 described below.



As outlined in the progressive manner in Figure 4, however, an institution may end up with the unfavorable consequence with potentially lower prestige, depending on (1) the location of the original allocation arrangement, (2) the extent to which the institution is able to reallocate its resources, that is, the institutional mobility, and (3) the shape of the prestige curve. In fact, this readjustment process by an institution is also drawn as part of Theorem 2 described below.

Assume that a new institution of higher education with  $N$  academic departments or functional activities begins operation with a given amount of originally endowed resources. As the institution continues its operation each year with an intended goal of maximizing its prestige or reputation through optimally allocating its limited resources, where will the institution find itself eventually? The authors prove that the institution settles in a convergent manner with one of the three particular points, depending on the initial state.<sup>9</sup>

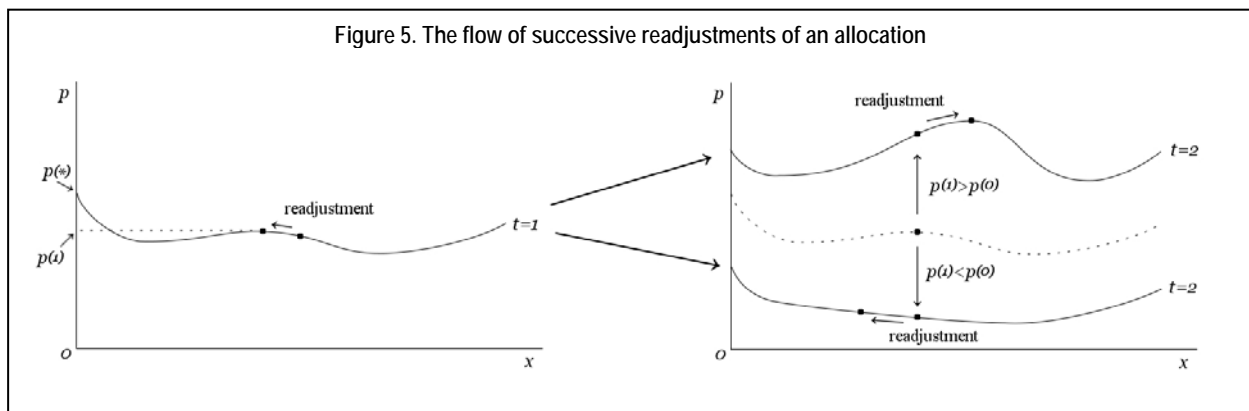
**Theorem 2.** *An institution convergently reaches the management point that may be described as (i) the point where all the original  $N$  functional activities are sustained; (ii) the point where some institutional activities are sustained while others are abolished; or (iii) the point where the institution ceases operation completely.*

In our model, it is assumed that the financial resources granted to an institution in a fiscal term is generally determined based on the overall prestige demonstrated by the institution in the previous term. The institution then determines an allocation arrangement for the current period so as to maximize its prestige with the currently available resources. The budget in the following period is again determined based on the resulting total prestige in the current period. If the institutional prestige achieved in the current term is assessed higher than the prestige attained in the previous period, then the total resources awarded in the next period will be increased, and so forth. The institution is assumed to repeat this process iteratively in subsequent years.

An anecdotal scenario goes as follows: imagine a case where a new college or university is founded in year  $t = 0$ . It is inevitable for this new institution to, perhaps through trial and error, investigate the appropriate institutional scale, e.g., sizes of student enrollment, faculty and administrative staff, as well as its scope, e.g., what and how many functional activities it should offer as an institution of higher education. However, no matter how carefully such a market investigation is conducted, it is extremely difficult for an institution to find the appropriate scale and functional scope that perfectly meet the market demand (i.e., demand from stakeholders which include prospective students, parents, local communities, local and central governments) in the beginning year.

Therefore, the institution continuously readjusts its organizational size and scope to fit to the market demand afterwards. Theorem 2 describes the potential consequences of such a readjustment process taken by an institution and guarantees that the process convergently ends at some point at which the state of institutional management becomes eventually stabilized with no further required adjustments, as it usually is the case in reality. Theorem 2 also predicts that the resulting outcome may not necessarily be a desirable one for the institution, however. That is, the institution may inevitably end up with the situation which requires reduction of its scale and scope by closing one of more functional activities (*i.e.*, the case (ii)), or in the worst scenario, it may be forced to shut down completely (*i.e.*, the case (iii)) being unable to find an optimal allocation arrangement and correspondingly stable prestige.

The typical flowing process of readjustments predicted for an institution is graphically presented in Figure 5 for the case with two functional activities. The horizontal and vertical axes in the figure represent allocation arrangement and the correspondingly achievable prestige, respectively. The left-side of Figure 5 shows the readjustment process in the current year ( $t = 1$ ). As a result of readjusted allocation in search of higher prestige (perhaps locally due to the rigidity), the institution achieves the overall prestige  $p^{(1)}$ . If this prestige is assessed higher than the total prestige achieved in the previous year  $p^{(0)}$ , that is,  $p^{(1)} > p^{(0)}$ , then a larger amount of institutional funding is awarded in the following year ( $t = 2$ ) than the current period, and the possible prestige curve in the next period ( $t = 2$ ) will be given by the upper curve in the right-side of Figure 5. Conversely, in the case  $p^{(1)} < p^{(0)}$ , the total funding granted in the next period decreases, and the possible prestige curve will be given by the lower curve in the same figure. In both cases, the institution continuously readjusts the allocation ratio between two activities in an attempt to find the highest total prestige.



Theorem 2 seems to present a quite obvious and naive result with no implicational importance. However, there exist at least two significant observations related to the second theorem. Firstly, the authors demonstrate that continually readjusted arrangement of internal resource allocation does not necessarily lead an institution to the optimizing management state which yields the potentially highest prestige. A resulting state may be given by only “locally” maximized prestige in a sense that the highest prestige is sought only within the limited range of allocation arrangements that are mutually agreed by the involved departments and divisions. For example, in the left-side of Figure 5, the highest prestige  $p^{(*)}$  can be achieved by concentrating its resources towards one end. However, such a drastic shift in reallocating resources may not be possible due to internal rigidity or the confined mobility as described previously. The institutional rigidity or the “stickiness of the adjustment process” is also pointed out by James (1990), Johnson and Turner (2009) and Massy (1996), attributing its cause to the conflicting interests of the involved parties competing in a zero-sum game within the same institution. Alternatively, the resulting state may be reached at a corner solution, at which one of the academic departments or schools must be closed in desperate efforts to improve institutional “quality” and “cost effectiveness”. Although complete closure of schools or departments is not observed on a daily basis, numerous cases may be found worldwide.<sup>10</sup>

Secondly, and perhaps more importantly in the policy aspect of higher education finance, Theorem 2 assumes that the social environment and conditions, based on which stakeholders determine how much to invest to an institution, remain constant throughout the entire readjustment process by the institution. Since drastic changes in such fundamental social conditions accompany the corresponding modifications in the amount of resources offered to an institution, it is a matter of course that the optimal allocation point at which a stable management state is reached for the institution will also shift eventually. In fact, such fundamental shifts by the stakeholders are often caused exogenously as a result of changing social conditions. A typical example is given by a situation where the funding criterion by the governments to each institution is affected due to volatile economic conditions and resulting changes in the financing policy of higher education. Such changes in government policies ordinarily intend to bring institutions to a particular state which is modified from the present state. Theorem 2 provides an analytical foundation on which whether such a policy intention is realized or not is objectively evaluated. That is, the effect of changing funding policies can

be assessed by examining the difference in reached states before and after the change occurred. In fact, Abe and Watanabe (2012b) theoretically investigate the varying effects of different funding schemes on the level of functional differentiation an institution can achieve.

### Conclusion

The primary objective of this paper is aimed at presenting geometric interpretations of a theory of optimal resource allocation with regard to prestige maximizing behaviors of a college or university, to a wider audience of students, practitioners, and scholars whose research interests cover the field of higher education finance. The series of theoretical developments by the authors is certainly a new emergence in the field of economics of higher education, and we believe that the currently highly debated issues of university rankings and prestige acquisition process could be discussed in a systematic and unambiguous manner with solid understanding of the mechanism which regulates prestige-seeking behaviors and resource allocation tactics of an institution of higher education.

As demonstrated in this geometrically interpreted version of the original work, the proposed theoretical framework with analytical tools help us examine the relationship between deliberately allocated resources and correspondingly acquired institutional prestige. The original framework, though requiring relatively advanced knowledge of mathematics, also provides an analytical foundation to evaluate the effectiveness of the changes in funding policies and schemes, which certainly presents significant implications to individual institutions as well as the involved stakeholders, particularly given the strictly tight fiscal environments surrounding today's higher education institutions worldwide.

Clearly, this paper does not explain all the components and results that are discussed in the original study, and some caveats still remain with the proposed theory for an improved coverage of a wider range of numerous actual cases as well as with substantiating the validity of theoretical findings with the existing data. Moreover, the model still requires further refinement to address and incorporate added complexities in reality. Such ambitious probes are yet to be conducted by the authors, and new innovative and theoretical discoveries are awaited.

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### ENDNOTES

1. Clearly, internally assessed prestige is not necessarily the same as externally evaluated prestige or reputation, due to measurement errors and asymmetric information existing between the institution and its stakeholders. Nonetheless, the model assumes, at least at the beginning of each fiscal term, that the institution allocates the available resources to its best knowledge in seeking the highest potential prestige. The authors then incorporate an error-correcting adjustment process to allow the objectively assessed feedback, which is examined as part of the proof of Theorem 3 in the original study. However, the formal presentation of this process requires mathematical treatments.
2. This naturally leads to the importance of further examinations of functional or mission differentiation among multiple university campuses. The authors, based on the same analytical model and diagnostic tools, also study the potential relationship between specific institutional funding schemes and the extent to which functional differentiation is attained by institutions of higher education (Abe and Watanabe, 2012b).
3. The proposed theoretical model intends to provide an equally valid analytical framework for examining the optimizing behaviors of not-for-profit organizations, such as hospitals, museums, and charitable and research organizations who also seek to gain prestige or social recognition through their activities, provision of which tends to fall short due to the nonprofit nature. (Frank and Salkever, 1991; Horwitz and Nichols, 2009; Newhouse, 1970). We believe that severe market rivalry is rather an unlikely assumption for many of these non-profit organizations.
4. The conflicting interests within a higher education institution are also discussed by James (1990) and Massy (1996) for different departments competing in a zero-sum game with the faculty trying to increase the prestige of only their particular department rather than the overall prestige of the institution. Moreover, Johnson and Turner (2009) attribute differences across academic fields in the allocation of tenure-track or tenured faculty to political forces within institutions which causes the "stickiness of the adjustment process." The consequence of such internal rigidity is examined in detail in the original work.
5. The authors also examined a case in which a different shape is assumed for the prestige function rather than imposing the "S-shaped" curve as depicted in Figure 1. For example, the authors consider a typical "strictly concave" shape, for which  $0 < \frac{dp_i}{dx_i}(0) < \infty$  and the increments along the positive slope continuously decelerate over the domain  $0 < x_i < \infty$ . It is demonstrated by the authors that all the results obtained in this paper remain valid for this more restrictive case with a strictly concave functional form of prestige.
6. Certainly, this "additively separable" functional form of institutional prestige is not an appealing feature, particularly if different academic disciplines, *e.g.*, economics and physics, contribute through joint efforts to the production of new knowledge which in turn leads to an increase of overall institutional prestige. The effect of interdisciplinary efforts across different academic departments or disciplines is considered in Abe and Watanabe (2012c).
7. In economics, the term "*iso*" simply means "equal", so the "*iso*-prestige" line may be interpreted as a line representing the equal or constant level, everywhere on the same line, of overall prestige ( $\bar{P}$ ) obtained with varying combinations of  $(p_1, p_2)$ .



8. In fact, this is a slight stretch from our original pledge of conducting “two-dimensional” analysis as the first scenario applies to an institution with three or more functional activities, *e.g.*, student instruction, faculty research, and community services. The first scenario describes a situation in which the institution which is unable to find the optimal allocation arrangement by managing three different functional activities, could indeed find the prestige maximizing allocation if it decided to focus only on two functions, *e.g.*, by shutting down the research activities.
9. Theorem 2 presented in this paper is the reinterpretation of Theorem 3 from the original study (Abe and Watanabe, 2012a). The original second theorem, which was derived as a by-product of Theorem 1, is omitted in this paper as it provides less significant implications than the other two. The original second theorem presents a mere fact that an institution endowed with abundant financial wealth is able to manage all the functional activities in a well-balanced way. It is worth noting, however, that this natural result is essentially derived from the “diminishing returns” in our framework, which means that the marginal prestige gain in each discipline or field steadily declines as the amount of funding provided to these fields increases. With this property, an institution would not effectively benefit from overly allocating a large amount of resources into a single department or major as the prestige of an academic department eventually reaches the plateau as shown in Figure 1. The institution could rather attain higher institutional prestige by allocating the excess resources to other fields and activities from which the institution is able to collectively and cost-effectively earn higher overall prestige.
10. Furnham and Simester (2000) show that the British public’s perceptions, in a cutting-saving exercise of academic departments, rated a certain group of academic disciplines with least importance, while several others are rated as “worthy of saving”. The result signifies the importance of influential perceptions given by stakeholders.

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